**KERALA UNIVERSITY**

Model Question Paper- M. Sc. Examination

Branch : Mathematics

MM 224-Partial Differential Equations and Integral Equations

Time: 3 hours Max. Marks:75

## Part A

***Answer any 5 questions from among the questions 1 to 8***

**Each question carries 3 marks**

1. Solve the equation $u_x = 1$ subject to the initial condition $u(0, y) = g(y)$.
2. Find the general solution to the equation $yu_x + xu_y = 0$ using the Lagrange method.
3. Let $u(x, t)$ be a solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$, which is defined in the whole plane. Assume that $u$ is constant on the line $x = 2 + ct$. Prove that $u_t + cu_x = 0$.
4. Consider the equation $u_{xx} - 2\sin x\ u_{xy} - \cos^2 x\ u_{yy} - \cos x\ u_y = 0$. Find a coordinate system $s = s(x, y), t = t(x, y)$ that transforms the equation into its canonical form.
5. Prove the necessary condition for the existence of a solution to the Neumann problem.
6. Determine the resolvent kernel associated with $K(x, \xi) = x\xi$ in the interval $(0, 1)$.
7. Using Euler's equation, find the shortest curve joining two points $(x_1, y_1)$ and $(x_2, y_2)$.
8. Find the point on the plane $ax + by + cz = d$ that is nearest the origin by the method of Lagrange multipliers. . $5 \times 3 = 15$

## Part B

***Anwer all questions from 9 to 13***

***Each question carries 12 marks***

9. A. a. Find a function $u(x, y)$ that solves the Cauchy problem $x^2 u_x + y^2 u_y = u^2, u(x, 2x) = x^2, x \in \mathbb{R}$.

   b. Check whether the transversality condition holds.

   **OR**

   B. a. Solve the equation $(y + u)u_x + yu_y = xy$ subject to the initial conditions $u(x, 1) = 1 + x$.

   b. Show that the Cauchy problem $u_x + u_y = 1, u(x, x) = 1$ is not solvable.

10. A. a. Prove that the equation $x^2 u_{xx} - 2xyu_{xy} + y^2 u_{yy} + xu_x + yu_y = 0$ is parabolic and find its canonical form and the general solution on the half-plane $x > 0$.

    b. Solve the problem

$$\begin{aligned} u_{tt} - u_{xx} &= t^7, & -\infty < x < \infty, t > 0 \\ u(x, 0) &= 2x + \sin x, & -\infty < x < \infty \\ u_t(x, 0) &= 0, & -\infty < x < \infty \end{aligned}$$

**OR**

B. Obtain the D'Alembert's solution of the following one dimensional wave equation

$$u_{tt} - c^2 u_{xx} = 0, \quad -\infty < x < \infty,\ t > 0$$
$$u(x,0) = f(x), u_t(x,0) = g(x), \quad -\infty < x < \infty$$

11. A. a. Solve the equation $u_t = 17u_{xx}, 0 < x < \pi, t > 0$, with the boundary conditions $u(0,t) = u(\pi,t) = 0, t \geq 0$ and the initial conditions

$$u(x,0) = \begin{cases} 0, & 0 \leq x \leq \frac{\pi}{2} \\ 2, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

b. State and Prove the Maximum principle.

**OR**

B. State and Prove The mean value principle. Is the converse true? Justify.

12. A. Find the characteristic values and characteristic functions for the equation $y(x) = \lambda \int_0^{2\pi} \sin(x+\xi)\ y(\xi)\ d\xi.$

**OR**

B. a. Form the Volterra equation corresponding to the initial value problem $y'' + xy = 1$ with $y(0) = y'(0) = 0$.

b. Show that the characteristic values of a Fredholm equation with a real symmetric kernel are all real.

13. A. Find the extremals for the integral $I = \int_{x_1}^{x_2} f(x,y,y')\ dx$ , if the integrand is

a. $y^2 - (y')^2$

b. $\dfrac{\sqrt{1+(y')^2}}{y}$

**OR**

B. A uniform flexible chain of given length hangs between two points. Find its shape if it hangs in such a way as to minimize its potential energy.

. $5 \times 12 = 60$