KERALA UNIVERSITY
Model Question Paper- M. Sc. Examination
Branch : Mathematics
MM 223- Toplogy-II

Time: 3 hours | Max. Marks:75

Part A
Answer any 5 questions from among the questions 1 to 8
Each question carries 3 marks

1. Prove that the projection maps $P_i : X \longrightarrow X_i$ where $X = X_1 X X_2 X....X_n$ are continous.
2. Prove that space X is a T_1 space if and only if each finite subset of X is closed
3. Prove or disprove " The set of dyadic numbers is dense in $\mathbb{R}$
4. Prove or disprove " The product of two normal spaces is normal".
5. In a product space XxX the set (x, x) is called the diagonal. Prove that X is Hausdorff if the diagonal set is closed
6. If $\mathfrak{F}$ is an unltrafilter on X and $f : X \longrightarrow Y$ is onto Y then show that $f(\mathfrak{F})$ is an ultrafilter on Y
7. Prove that a convex set A in R^n is contractible to each point x_0 in A
8. .Is the unit circle S^1 a retract of the closed unit ball B^2 ? Justify your claim.

Part B
Answer all questions from 9 to 13
Each question carries 12 marks

9. A. a) . Prove that product of a finite number of compact spaces is compact 6
   b) Prove that the product of an arbitrary collection of connected spaces is connected. 6

OR

   B. a) Prove that a product of a finite number of second countable spaces is second countable 5
   b) Show that the cantor set C is homeomorphic to a countable infinite product of two point spaces 7

10. A State and prove Tietze Extension theorem 12

OR

   B a) Show that every metric space is normal 5
   b) If X is a separable normal space and E a subset of X with $card\ E \geq c$, show that E has a limit point in X. 7

11. A State and prove Tychonoff theorem. 12

OR

B a) Prove that every filter is contained in an ultra filter. 6

b) Prove that a topological space is Hausdorff iff limits of all nets in it are unique. 6

12. A. State an prove Covering path property. 12

OR

B. a) Prove that the fundamental group of S^1 is isomorphic to the additive group $\mathbb{Z}$ of integers. 6

b) Show that every contractible space is simply connected. 6

13. A. For $n \geq 2$, show that S^n is simply connected. 12

OR

B. a) If D is a deformation retract of space X and x_0 is a point of D, show that $\prod_1(X, x_0)$ and $\prod_1(D, x_0)$ are isomorphic. 8

b) Find the fundamental group of a cylinder C. 4