KERALA UNIVERSITY

Model Question Paper- M. Sc. Examination

Branch : Mathematics

Course Code-222: Course Name: Real Analysis II

Time: 3 hours Max. Marks:75

Part A

Answer any 5 questions from among the questions 1 to 8.

Each question carries 3 marks.

1. Define Lebesgue Outer measure . Find the Lebesgue Outer measure of a singleton set.
2. Define Lebesgue measurable function. Give one example.
3. Define Lebesgue integral of a simple measurable function. Find the Lebesgue integral of $f(x)=1; 0\leq x\leq 4$.
4. Define upper right, upper left, lower right and lower left derivatives of an extended real valued function f(x) at x.
5. Define a complete measure. Give one example
6. Give one example of a convex function on [0,1] which is not continuous.
7. State Radon –Nikodym Theorem.
8. Let f be non-negative measurable function such that $\int f dx=0$. Prove that $f=0\, a.e$

Part B

Answer all questions from 9 to 13

Each question carries 12 marks

9.

A. Prove that Lebesgue outer measure of an interval is its length.

OR

B. a) Show that there exist an uncountable set of zero measure

b) Prove that there exist a non-measurable set

10.

A. a) Let f be a non-negative measurable function . Prove that there exist a sequence $\{\varphi_n\}$ of simple measurable functions such that, for each x , $\varphi_n(x)$ is increasing and converge to $f(x)$.

b) Show that $\int_1^\infty \frac{1}{x} dx = \infty$

OR

B. a) State and prove Lebesgue Dominated Convergence Theorem.

b) Show that if f is integrable, then f is finite-valued $a.e$.

11.

A) Prove that Every σ-finite measure on a ring R has a unique extension to the σ-ring $S(R)$.

OR

B. a) State and prove Fatou's Lemma.

b) Give an example where strict inequality occurs in Fatou's Lemma.

12.

A. State and prove Jensen's inequality.

OR

B. a) Define convex function.

b) Prove that every convex function defined on an open interval is continuous.

13.

A) Prove that every signed measure v on a measurable space $[X, S]$ has a Jordan Decomposition.

OR

B) Prove that every signed measure v on a measurable space $[X, S]$ has a Hahn Decomposition.