KERALA UNIVERSITY

Model Question Paper – M.Sc Examination 2020 Admission onwards

Branch: Mathematics

MM 221-ABSTRACT ALGEBRA

Tme: 3 hours Max: 75

Part A

Answer any 5 question from among the question 1 to 8.

Each question carries 3 marks

1.Determine the number of elements of order 5 in $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$.

2.Find the number of Abelian groups up to isomorphism of order 42.

3.Let G be noncyclic group of order 21.Find the number of Sylow 3– subgroups of G.

4.Show that there is no simple group of order 216.

5.Find the splitting field of $x^4 + 1$ over the set of rational numbers `Q.

6.If E is a finite extension of F, then show that E is an algebraic extension of F.

7.Draw subfield lattice of GF(2^{24}).

8.Factor $x^6 - 1$ as a product of irreducible polynomials over $\mathbb{Z}_2$.

Part B

Answer all questions from 9 to 13

Each question carries 12 marks

9.

A. a)Determine the number of cyclic subgroups of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$.

b)State and prove $G/\mathbb{Z}$ theorem.

B. a)If a group G is an internal direct product of a finite number of subgroups $H_1, H_2, \ldots, H_n$,

then prove that G is isomorphic to the external direct product of $H_1, H_2, \ldots, H_n$.

b) Does every Abelian group of order 45 have an element of order 9.Justify your answer.

10.

A. a) State and prove Sylow's second theorem.

b)G is any group of order 99.Then show that G is isomorphic to $\mathbb{Z}_{99}$ or $\mathbb{Z}_3 \oplus \mathbb{Z}_{33}$.

B. a) State and prove generalized Cayley theorem.

b)Show that A_5 does not contain a subgroup of order 30,20, or 15.

11.

A. a) Let F be a field and f(x) be anon constant polynomial in F[x]. Then prove that there is an extension field E of F in which f(x) has a zero.

b) Define perfect field. Prove that every finite field is perfect.

B. a)Let K be a finite extension field of the field E and let E be a finite extension field of the field F. Then K is a finite extension field of F and [K:F]= [K:E][E:F].

b)Show that $Q(\sqrt{3}, \sqrt{5}) = Q(\sqrt{3} + \sqrt{5})$

12.

A. a) Prove that for each divisor m of n , GF(P^n)has a unique subfield of order p^m.More over ,these are the only subfield of GF(p^n).

b) Let α be a zero of f(x)= $x^2 + 2x + 2$ in some extension field of $\mathbb{Z}_3$. Find the other zero of f(x) in

B. a)Prove that as a group under addition ,Gf(p^n) is isomorphic to $\mathbb{Z}_P \oplus \mathbb{Z}_p \oplus \cdots \mathbb{Z}_P$, n factors and as a group under multiplication, the set of non zero elements of GF(p^n) is isomorphic to $\mathbb{Z}_{p^n_{-1}}$.

b) Prove that sinθ is constructible if and only if cosθ is constructible .

13.

A a)Define Galois group. Let F be a field of characteristic 0 and let aϵF. If E is the splitting field of x^n-a over F , then prove that the Galois group Gal(E/F) is solvable.

b)Prove that factor group of a solvable group is solvable.

B. a) Prove that the cyclotomic polynomial $\emptyset_n$(x) are irreducible over Z.

b) Let ω be a primitive nth root of unity. Then Gal$(Q(\omega)/$Q$)$ $\approx$ U(n).